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CONCERNING THE CONJUGATE OF A PARTITION

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Abstract: We develop an algebraic formula for the conjugate of a partition. As an immediate consequence, we obtain an alternate proof for the known result that the number of distinct parts of a partition is invariant under conjugation. In addition, we present a theorem concerning the multiplicities of the parts of a partition.

Keywords and Phrases: Conjugate partition, distinct partition.

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1. Introduction

Let λ be a partition of the natural number n specified by

$$n = n_1 + n_2 + n_3 + \cdots + n_r \tag{1}$$

where the n_i are natural numbers such that

$$n_i \geq n_{i+1} \quad for\ all\ i. \tag{2}$$

Note that r represents the total number of parts in λ. The Ferrers graph of λ is a left-justified array consisting of n_i dots in the i^{th} row, where $1 \leq i \leq r$. This graph contains columns as well as rows. The *conjugate* of λ, denoted λ^*, is the partition obtained by interchanging the rows and columns of the Ferrers graph of λ. (This operation can also be called reflection about the main diagonal.) For an alternate definition of conjugate partition, see [4], Definition 1.8 on p.7 .

In this note, we obtain a formula for λ^*. We also show that the number of distinct parts of a partition is invariant under conjugation. This result has been previously stated, but not proven, by K. Alladi. (See [1], [2].) We also mention a simple proof